QM Spring 2019: HW 4

1. Consider the 2×2 matrix Hamiltonian

$$H = H_0 + \epsilon \widehat{n}^i \sigma^i, \tag{0.0.1}$$

where σ^i is the ith Pauli matrix and

$$H_0 = \begin{bmatrix} \bar{E} + \delta & 0 \\ 0 & \bar{E} - \delta \end{bmatrix}, \tag{0.0.2}$$

$$\widehat{n}(\phi) = (\cos\phi, \sin\phi, 0)^{\mathrm{T}}. \tag{0.0.3}$$

- Assume $\delta > 0$ is fixed and use non-degenerate perturbation theory to solve the eigenvalues of H up to order ϵ.
- Set $\delta = 0$. Use degenerate perturbation theory to compute the eigenvalues of H up to order ϵ.
- Diagonalize H exactly. Using these exact eigenvalues of H: take the limit $\epsilon \to 0$ assuming δ is fixed and compare the result of taking $\delta \to 0$ followed by expanding the result up to order ϵ. Comment on the difference.

2. Weinberg 5.1.

3. Weinberg 5.2.

4. Weinberg 5.4.

5. Verify that the states given in Weinberg eq. (5.3.10) do indeed diagonalize $\delta H = e\vec{X} \cdot \vec{E}$ from the first order Stark effect.

6. *From L. Pauling and E. B. Wilson* Consider the system consisting of two valence electrons (Helium, for example); with potential given by

$$V = -\frac{Ze^2}{|\vec{x}_1|} - \frac{Ze^2}{|\vec{x}_2|} + \frac{e^2}{|\vec{x}_1 - \vec{x}_2|}. \tag{0.0.4}$$

Neglecting the $e^2/|\vec{x}_1 - \vec{x}_2|$ term for the moment, solve the ground state eigenket and eigen-energy of the Hamiltonian

$$H_0 = \frac{\vec{P}_1^2 + \vec{P}_2^2}{2m_e} - \frac{Ze^2}{|\vec{x}_1|} - \frac{Ze^2}{|\vec{x}_2|}. \tag{0.0.5}$$

Now treat the $e^2/|\vec{x}_1 - \vec{x}_2|$ term as a perturbation – i.e., define

$$\delta_1 H \equiv \frac{e^2}{|\vec{x}_1 - \vec{x}_2|}. \tag{0.0.6}$$

Compute the first order corrections to the ground state energy of Helium in this framework. Hint: You may need the expansion

$$\frac{1}{|\vec{x}_1 - \vec{x}_2|} = \frac{4\pi}{r_>} \sum_{\ell=0}^{+\infty} \frac{(r_</r_>)^\ell}{2\ell+1} \sum_{m=-\ell}^{+\ell} Y_\ell^m(\widehat{x}_1) \overline{Y_\ell^m(\widehat{x}_2)}, \tag{0.0.7}$$

$$r_> \equiv \max(|\vec{x}_1|, |\vec{x}_2|), \qquad r_< \equiv \min(|\vec{x}_1|, |\vec{x}_2|). \tag{0.0.8}$$

□