# QM Spring 2019: HW 1

1. Weinberg 4.7.
2. Weinberg 4.8.
3. Weinberg 4.9.
4. If $Y_\ell^m$ is a spherical harmonic and if $|\vec{X}|^\ell Y_\ell^m(\widehat{X})$ is viewed as an operator built out of the position operator $\vec{X}$, explain why $Y_\ell^m(\widehat{X})$ is a tensor operator. Use the Wigner-Eckart theorem to evaluate the following solid angle integral on the 2−sphere in terms of Clebsch-Gordan coefficients:
$$\int_{\mathbb{S}^2} \overline{Y_{\ell_1}^{m_1}(\widehat{x})} Y_{\ell_2}^{m_2}(\widehat{x}) Y_{\ell_3}^{m_3}(\widehat{x}) \mathrm{d}\Omega_{\widehat{x}}. \tag{0.0.1}$$
5. If $J^i$ and $V^i$ are the Cartesian components of, respectively, the angular momentum and some vector operator, namely Weinberg eq. (4.4.5) is obeyed – verify that his equations (4.4.7) and (4.4.8) are satisfied by the operators defined in eq. (4.4.6). Explain why, this in turn allows us to identify
$$O_1^{\pm 1} \equiv V^{\pm} \qquad \text{and} \qquad O_1^0 \equiv V^3. \tag{0.0.2}$$
6. Consider two spin-half particles in a bound system, such that the total Hamiltonian can be written in terms of their reduced mass $\mu$; relative coordinate $\vec{X}$; relative orbital angular momentum $\vec{L}$; their individual and total spin operators $\vec{S}'$, $\vec{S}''$ and $\vec{S} \equiv \vec{S}' + \vec{S}''$; as well as the total angular momentum $\vec{J} \equiv \vec{L} + \vec{S}$:
$$H = -\frac{1}{2\mu}\left\{\frac{1}{r^2}\partial_r\left(r^2\partial_r\cdot\right) - \frac{\vec{L}^2}{r^2}\right\} + V_0(r) + V_1(r)\left(\vec{S}'\cdot\vec{S}''\right) + V_3(r)\left(\vec{L}\cdot\vec{S}\right), \tag{0.0.3}$$
$$r \equiv |\vec{x}|. \tag{0.0.4}$$
Use the separation-of-variables technique – i.e., assume the wave function is a function of $\vec{x}$ multiplied by some spin-dependent state – and write down the ordinary differential equation for the energy eigenstate with total angular momentum $j$, total orbital momentum $\ell$ and total spin $\sigma$.[1] Hint: First explain why
$$\vec{S}'\cdot\vec{S}'' = \frac{1}{2}\left(\vec{S}^2 - \vec{S}'^2 - \vec{S}''^2\right), \tag{0.0.5}$$
$$\vec{L}\cdot\vec{S} = \frac{1}{2}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right). \tag{0.0.6}$$

[1] This problem can be found in at least one quantum mechanics text – which shall be revealed in the solutions – so make sure you explain your steps carefully!