QM Spring 2020: In-Class Presentation

1. **Landau Levels** Suppose a constant magnetic field points in the 3−direction:
$$\vec{B} = B\hat{z}, \qquad B > 0. \tag{0.0.1}$$
First assume this $\vec{B}$ field fills all of space. Solve for the eigenstates and energy eigenvalues of the non-relativistic fermionic Hamiltonian
$$H = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 + g\vec{B}\cdot\vec{S}, \tag{0.0.2}$$
where m is the mass of the charged particle, e is its charge; $\vec{A}$ is the vector potential; and $\vec{S} \equiv \vec{\sigma}/2$ is the spin operator.

2. **Boundary Conditions** What happens when the B field does not fill the whole space? For example, consider solving the same problem but with an infinitely long solenoid of radius R, lying parallel to the 3−axis. You may let the walls of the solenoid be opaque or transparent to the particle; or even consider other geometries. Is there a limit where you recover the space-filling-B-field results above?

3. **Electromagnetic perturbation** Consider turning on a plane wave oscillating electromagnetic field; or an electric field within a slab, say, $0 \leq x^2 \leq L$, where x^2 is the 2−direction (and is hence perpendicular to $\vec{B}$). (You can invent your own type of electromagnetic field too.) How are the energy levels and eigen wavefunctions affected?

4. **Inter-electron interactions** Consider the two identical-body Hamiltonian
$$H = H_0 + H_1, \tag{0.0.3}$$
$$H_0 = \sum_{a=1}^{2}\left\{\frac{1}{2m}\left(\vec{p}_a - e\vec{A}(\vec{X}_a)\right)^2 + g\vec{B}\cdot\vec{S}_a\right\}, \qquad H_1 = -\frac{e^2}{4\pi|\vec{X}_1 - \vec{X}_2|}. \tag{0.0.4}$$
By treating H_1 as a perturbation, can you compute the eigen-energies up to first order in perturbation theory? Can you explain when perturbation theory breaks down?

5. **Quantum Statistical Mechanics (Bonus)** Suppose we have a (large number of) $N \gg 1$ *identical* fermions immersed in the constant space-filling $\vec{B}$ field described by eq. (0.0.1); and the Hamiltonian
$$H = \sum_{a=1}^{N}\left\{\frac{1}{2m}\left(\vec{p}_a - e\vec{A}(\vec{X}_a)\right)^2 + g\vec{B}\cdot\vec{S}_a\right\}. \tag{0.0.5}$$
Can you discuss the quantum statistical mechanics of such a system?