QFT HW 5

1. Compute the 2 point Euclidean correlator:

$$\langle \varphi(x_1)\varphi(x_2)\rangle \equiv \frac{\int \mathcal{D}\varphi \varphi(x_1)\varphi(x_2)\exp\left(-S_{\mathrm{E}}\right)}{\int \mathcal{D}\varphi \exp\left(-S_{\mathrm{E}}\right)}, \tag{0.0.1}$$

$$S_{\mathrm{E}} \equiv \int_{\mathbb{R}} \mathrm{d}x \left(\frac{1}{2}\varphi'(x)^2 + \frac{m^2}{2}\varphi(x)^2 + \frac{\lambda}{4!}\varphi(x)^4\right); \tag{0.0.2}$$

up to $\mathcal{O}(\lambda)$. Note that, even though this is a Euclidean QFT, the perturbation theory is similar to that in Lorentzian QFT. □

2. Consider the following Lagrangian density describing 2 massive and 2 massless fermions ($\psi_{1,2}$ and $\nu_{1,2}$ respectively); and their 4-fermion interaction:

$$\mathcal{L} = \sum_{\mathrm{I}=1}^{2} \left\{\overline{\psi}_{\mathrm{I}}\left(i\partial\!\!\!/ - m_{\mathrm{I}}\right)\psi_{\mathrm{I}} + \overline{\nu}_{\mathrm{I}} i\partial\!\!\!/ \nu_{\mathrm{I}}\right\}$$
$$+ \frac{g^2}{M^2}\overline{\psi}_1\gamma^\mu\left(c_1 - c_2\gamma^5\right)\nu_1\overline{\nu}_2\gamma_\mu\left(c_3 - c_4\gamma^5\right)\psi_2 + \mathrm{h.c.}, \qquad c_{1,2,3,4} \in \mathbb{R}. \tag{0.0.3}$$

Assuming $m_1 > m_2$, compute – to leading order in g^2– the decay rate of the process

$$\psi_1 \to \nu_1 + \psi_2 + \overline{\nu}_2 \tag{0.0.4}$$

due to the interaction in eq. (0.0.3). Muon and quark decays are special cases of this process. □