HW 9 Problems 1 and 2

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1 Problem 1

Complex Exponentials Write the following complex numbers in the form $x + iy$, where x and y are real.

- $\exp(i\pi/4)$
- $\sqrt{2}\exp(i\pi/3)$
- $\pi e^{i\pi/6}$

Write the following complex numbers in the form $re^{i\phi}$ where r and ϕ are real.

- $3 + 4i$
- $e + \pi i$
- $2 + \sqrt{3}i$

Define

$$P(t) \equiv P_0 e^{i\omega t + i\varphi_1} \qquad \text{and} \qquad Q(t) \equiv Q_0 e^{i\Omega t + i\varphi_2}, \tag{1.0.1}$$

where P_0, Q_0, ω, Ω, φ_1 and φ_2 are constants. Given constants α, β, γ and σ; show that

$$\alpha P'(t) + \beta Q'(t) = AP(t) + BQ(t), \tag{1.0.2}$$

$$\gamma P''(t) + \sigma Q''(t) = CP(t) + DQ(t). \tag{1.0.3}$$

Express A, B, C and D in terms of the constants in the problem. Hint: calculus for complex differentiable functions work like the real case – in particular, $(\mathrm{d}/\mathrm{d}z)e^z = e^z$ for complex z.

2 Problem 2

Critically damped RLC circuit Refer to Benson eq. 32.17 describing the RLC circuit.

$$L\frac{\mathrm{d}^2 Q}{\mathrm{d}t^2} + R\frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{Q}{C} = 0 \tag{2.0.4}$$

$Q(t)$ is the charge on the capacitor. Verify that

$$Q(t) = Q_0\, te^{-t/\tau} \tag{2.0.5}$$

solves eq. (2.0.4) when $1/\sqrt{LC} = R/(2L)$, where Q_0 is an arbitrary constant. What is τ?