Physics in Curved Spacetimes: Homework Problem

Problem 0.1. 'Flow' Induced By Irreducible Decomposition Physical problems often lead to the following matrix equation in (flat) space

$$\left(\frac{\mathrm{d}^2 x^i}{\mathrm{d}t^2} \text{ or } \frac{\mathrm{d}x^i}{\mathrm{d}t}\right) = \Sigma^{ij}(t) x^j(t). \tag{0.0.1}$$

To simplify the analysis, let consider this problem in 2D; i.e., x^i is the 2D Cartesian coordinate displacement vector, and Σ^{ij} is an arbitrary 2×2 matrix. We may then decompose

$$\Sigma^{ij} = \frac{\delta^{ij}}{2}\delta_{ab}\Sigma^{ab} + \frac{1}{2}\Sigma^{[ij]} + \left(\frac{1}{2}\Sigma^{\{ij\}} - \frac{\delta^{ij}}{2}\delta_{ab}\Sigma^{ab}\right). \tag{0.0.2}$$

Use a computer program to plot the vector field representation of its irreducible parts:

$$\delta^{ij} x^j; \tag{0.0.3}$$

$$\left(\widehat{e}_1^i \widehat{e}_2^j - \widehat{e}_2^i \widehat{e}_1^j\right) x^j; \tag{0.0.4}$$

$$\left(\widehat{e}_1^i \widehat{e}_1^j - \widehat{e}_2^i \widehat{e}_2^j\right) x^j, \qquad \left(\widehat{e}_1^i \widehat{e}_2^j + \widehat{e}_2^i \widehat{e}_1^j\right) x^j. \tag{0.0.5}$$

Also be sure to explain how these terms in equations (0.0.3)–(0.0.5) are related to eq. (0.0.2). □