Coordinates 2D and 3D

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1 2 Dimensions

In 2 dimensions, we may use Cartesian coordinates $\vec{r} = (x, y)$ and the associated infinitesimal area

$$\mathrm{d}A = \mathrm{d}x\mathrm{d}y. \tag{1.0.1}$$

The gradient of a scalar V is

$$\vec{\nabla}V = \partial_x V\widehat{x} + \partial_y V\widehat{y}. \tag{1.0.2}$$

We may also employ polar coordinates

$$\vec{r} = (x, y) = \rho\left(\cos\phi, \sin\phi\right), \tag{1.0.3}$$
$$\rho \geq 0, \qquad\qquad 0 \leq \phi < 2\pi. \tag{1.0.4}$$

The associated infinitesimal area is

$$\mathrm{d}A = \rho\mathrm{d}\rho\mathrm{d}\phi. \tag{1.0.5}$$

The gradient of a scalar is

$$\vec{\nabla}V = \partial_r V\widehat{r} + \frac{1}{r}\partial_\phi V\widehat{\phi}. \tag{1.0.6}$$

2 3 Dimensions

In 3 dimensions, we may use Cartesian coordinates $\vec{r} = (x, y, z)$ and the associated infinitesimal volume

$$\mathrm{d}V = \mathrm{d}x\mathrm{d}y\mathrm{d}z. \tag{2.0.1}$$

The gradient of a scalar V is

$$\vec{\nabla}V = \partial_x V\widehat{x} + \partial_y V\widehat{y} + \partial_z V\widehat{z}. \tag{2.0.2}$$

We may also employ cylindrical coordinates

$$\vec{r} = (x, y, z) = \left(r\cos\phi, r\sin\phi, z\right), \tag{2.0.3}$$
$$\rho \geq 0, \qquad\qquad 0 \leq \phi < 2\pi, \qquad\qquad z \in \mathbb{R}. \tag{2.0.4}$$

The associated infinitesimal volume is

$$dV = \rho d\rho d\phi dz. \tag{2.0.5}$$

The outward pointing area element on the curved surface of a cylinder of radius ρ is

$$d\vec{A} = \widehat{\rho} dA = \rho d\phi dz \widehat{\rho}. \tag{2.0.6}$$

The gradient of a scalar V is

$$\vec{\nabla} V = \partial_\rho V \widehat{\rho} + \frac{1}{\rho} \partial_\phi V \widehat{\phi} + \partial_z V \widehat{z}. \tag{2.0.7}$$

Spherical coordinates are defined as

$$\vec{r} = (x, y, z) = r(\sin\theta \cdot \cos\phi, \sin\theta \cdot \sin\phi, \cos\theta), \tag{2.0.8}$$

$$r \geq 0, \qquad 0 \leq \theta \leq \pi, \qquad 0 \leq \phi < 2\pi. \tag{2.0.9}$$

The associated infinitesimal volume is

$$dV = r^2 \sin\theta dr d\theta d\phi. \tag{2.0.10}$$

The outward pointing area element on the surface of a sphere of radius r is

$$d\vec{A} = \widehat{r} dA = \widehat{r} r^2 \sin\theta d\theta d\phi. \tag{2.0.11}$$

The gradient of a scalar V is

$$\vec{\nabla} V = \partial_r V \widehat{r} + \frac{\partial_\theta V}{r} \widehat{\theta} + \frac{\partial_\phi V}{r \sin\theta} \widehat{\phi}. \tag{2.0.12}$$